

Simple Model for the Anthropogenically Forced CO₂ Cycle Tested on Measured Quantities

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

The carbon dioxide information analysis center (CDIAC) provides a remarkable 163 years of data on atmospheric CO₂ concentrations, man-made CO₂ emissions, and from 1959 onwards CO₂ net-fluxes into oceans and biosphere. Currently, half of the anthropogenic CO₂ emissions remain in the atmosphere. Predominantly the ocean and the biosphere absorb the second half in about equal parts. We describe the anthropogenically forced CO₂ dynamics by a linear model of only two parameters which represent physics and biological laws. Our model reproduces the CDIAC measurements perfectly, and allows thus predictions for the future. It does not deal with the equilibrium exchanges of CO₂ between atmosphere, oceans and biosphere, but treats merely the net-fluxes resulting from the perturbation of the equilibrium by the anthropogenic emissions. Details as yielded by tracer measurements or ocean chemistry are not required. We applied the model for a tentative projection of the future CO₂ cycle based on prospective anthropogenic emission

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scenarios from the literature. As a result, the increase of atmospheric CO₂ will gradually come to an end and the ocean as well as the biosphere will be the primary sinks of future CO₂ emissions of mankind.

Keywords: Global carbon cycle; anthropogenic CO₂ emissions; linear circulation model; future atmospheric CO₂ concentrations.

1 INTRODUCTION

Before the industrialisation and considerable land use the ratio of CO₂ in the atmosphere and in the oceans had been stationary. At the beginning of the industrial era (1750 AD) the atmospheric CO₂ concentration was 277 ppm [1], corresponding to $2.12 \cdot 277 = 587.2$ GtC with 2.120 GtC/ppm as the correspondence of atmospheric carbon with CO₂ concentration [2]. The CO₂ content of the ocean is much higher, approximately 37 000 GtC [3].

Presently (2013), the atmospheric CO₂ concentration has risen to 395 ppm, or to an extra of $(395 \text{ ppm} - 277 \text{ ppm}) \cdot 2.12 \text{ GtC/ppm} = 250$ GtC, mainly due to fossil fuel burning, slash-and-burn of forests and cement production. The total anthropogenic CO₂ production is 10.7 GtC/yr or roughly 5 ppm/yr CO₂. About 2.5 ppm/yr of this quantity remains in the atmosphere, the rest is absorbed by the ocean and the biosphere in roughly equal amounts [4].

Since the year 1959 observations of atmospheric CO₂ contents and fluxes between atmosphere, ocean and biosphere have increased substantially. Most familiar among these is the atmospheric CO₂ content published presently in monthly updates by the Carbon Dioxide Information Analysis Center [4]. From the year 2006 on CDIAC established stocks of the major components of the global carbon budget that are the anthropogenic CO₂ emissions, the CO₂ fluxes into the atmosphere, the ocean and the biosphere. In the present paper we use the "Global Carbon Budget 14" which comprises the years 1959 - 2013. For the years before 1959 back to 1850 we use historical data of anthropogenic CO₂ emissions [4] and of atmospheric CO₂ concentrations [1].

Models of the carbon cycle under the forcing of anthropogenic CO₂ emissions have been publis-

hed among others by [5, 6]. Most model - work before 1970 is cited by [6]. More recently, 15 known carbon cycle models are compared on the response to a CO₂ impulse of 100 GtC in the year 2010 [7].

Modern models include the details of complex interactions between atmosphere, ocean and biosphere with their pertinent parameters. Among these are saturation of the ocean uptake under increasing atmospheric CO₂ concentrations, soil respiration, mixed atmospheric and oceanic multi-layers, divisions of the hemispheres into segments, and multiple time constants for the CO₂ exchange between atmosphere, ocean, and biosphere. The model parameters are obtained from observations, measurements – as for instance from the ¹⁴C/¹²C ratio [6] – and fitting procedures. However, no systematic comparison of the whole entity of the CDIAC observations with any CO₂ global circulation model has been published so far.

The results of atmospheric ¹⁴CO₂ measurements, which showed the interruption of the natural ¹⁴CO₂/¹²CO₂ ratio by the nuclear bomb test program between 1950 and 1963, yielded new insight in the CO₂ exchange between atmosphere and ocean. During this program the $\Delta^{14}\text{C}$ increased from -20 per mil to ~ 1000 per mil in the Northern hemisphere [8]. The rapid decrease of the ¹⁴CO₂ after the nuclear test ban treaty of the year 1963 has caused some confusion between the Residence Time "RT" and the Adjustment Time "AT" of an artificial CO₂ excess in the atmosphere.

The carbon cycle exchanges vast amounts of CO₂ between atmosphere, ocean and biosphere – roughly 90 GtC/yr in all directions (total fluxes). Due to the large total fluxes the RT of CO₂ of ~ 5 years in the atmosphere is relatively short. In contrast to this, the net-fluxes

from the atmosphere into the ocean and the biosphere, with only ~ 2.5 GtC/yr each, are much smaller and are caused by anthropogenic carbon emissions – except for minor natural fluctuations. The net-fluxes are essential for the AT to a new CO₂ equilibrium. As a consequence, the AT is more than an order of magnitude higher than the RT and independent of it [9],[10]. Both quantities RT and AT are generally in the literature given as "half-life".

In contrast to the many complex CO₂ circulation models which consider all CO₂ fluxes our objective was to model the anthropogenically forced CO₂ cycle by a minimum of physical assumptions considering exclusively the carbon net-fluxes. We do not model the detailed processes of the total fluxes but simply determine the only two relevant parameters of our linear Ansatz from the CDIAC data. Details of the cycle such as tracer results, biogenic ocean dynamics and chemistry as explored by complex and extensive models and belonging to the equilibrium state of the total CO₂ fluxes are beyond the scope of this work and unnecessary for such a simple model.

Following the aim of simplification, we use only one parameter for the atmospheric-oceanic net-flux of CO₂ and another one for the complex interaction of the biosphere with the atmospheric CO₂. The validity of the model is verified by comparison with the data [4]. As a model input we use only the anthropogenic CO₂ emissions from 1850 until present. We evaluate and compare the CO₂ remaining from a hypothetical CO₂ impulse of 100 GtC in the year 2010 of our model with [7].

2 THE MODEL

In the following we generally use carbon quantities and fluxes instead of CO₂ quantities. The already mentioned factor 2.12 GtC / ppm yields the conversion between both. For clarity, carbon fluxes $n(t)$ are generally written in small and their integrated values $N(t)$ in capital letters. All carbon fluxes in our model are net-fluxes.

Our model makes only two assumptions that are based on the linear chemical law of mass action:

Firstly, the carbon flux between atmosphere and ocean $n_s(t)$ is proportional to the difference of the actual and the preindustrial atmospheric CO₂ concentration yielding

$$n_s(t) = 1/\tau \cdot (N_a(t) - N_0) \quad (2.1)$$

with $N_a(t)$ the carbon content of the atmosphere in the year t , the constant carbon content $N_0 = N_a(1750) = 587.2$ GtC as the already mentioned value in the year 1750, and $1/\tau$ [yr^{-1}] as parameter of the net CO₂ uptake process by the ocean. Our assumption of a constant N_0 is based on a negligible change of the oceanic carbon amount because the ocean contains roughly the fortyfold dissolved carbon of the atmosphere. We note that the model neither contains the much higher mutual CO₂ exchange between ocean, atmosphere and biosphere nor the complex biogenic activity in the oceans nor the ocean chemistry. Similar simple assumptions as Eq. (2.1) have already been given by [11],[12],[13].

In a second assumption, biospheric increase $n_b(t)$ far from saturation is estimated as proportional to the atmospheric carbon increase $n_a(t)$

$$n_b(t) = b \cdot n_a(t) \quad (2.2)$$

with $n_a(t) = dN_a(t)/dt$ the carbon flux into the atmosphere, $n_b(t) = dN_b(t)/dt$ the carbon flux into the biosphere, and b the parameter of the process. This simple approximation corresponds to the known effect of better plant growth due to higher atmospheric CO₂ concentrations [14],[15],[16]. Our model of the Eqs. (2.1, 2.2) is linear.

In the following, bars are used for observed quantities for distinction from model quantities. Together with the anthropogenic carbon emissions $\bar{n}_{tot}(t)$ and Eq. (2.2) the sum rule

$$\bar{n}_{tot}(t) = (1 + b) \cdot n_a(t) + n_s(t) \quad (2.3)$$

and the equivalent sum rule for the integrated quantities

$$\bar{N}_{tot}(t) = N_a(t) + N_b(t) + N_s(t) \quad (2.4)$$

hold where n_s , n_b and n_a are defined in Eq. (2.1), Eq. (2.2). Eqs. (2.1) - (2.3) and $n_a = dN_a/dt$ can

be combined to

$$\frac{dN_a(t)}{dt} = \frac{1}{1+b} \left[\bar{n}_{tot}(t) - \frac{1}{\tau} (N_a(t) - \bar{N}_0) \right] \quad (2.5)$$

$N_a(t)$ in Eq. (2.1) has to be completed with a temperature term because the equilibrium between oceanic and atmospheric partial CO₂ pressures shifts slightly with sea temperature,

$$\bar{S}_a(t) = \mu \cdot 2.12 \cdot \bar{T}(t) = 15.9 \cdot \bar{T}(t) \quad (2.6)$$

$\bar{S}_a(t)$ [GtC] is the amount of carbon released into the atmosphere or absorbed by the ocean caused by changing temperatures, $\bar{T}(t)$ [°C] the average Earth temperature [17] converted to an anomaly around the 1850 AD value, μ the CO₂ production coefficient given as $\mu = 7.5$ ppm / °C [1], and 2.12 GtC/ppm the already mentioned ratio of atmospheric carbon to CO₂ concentration. This completes the first order differential equation Eq. (2.5) to

$$\frac{dN_a(t)}{dt} = \frac{1}{1+b} \left[\bar{n}_{tot}(t) - \frac{1}{\tau} (N_a(t) + \bar{S}_a(t) - \bar{N}_0) \right] \quad (2.7)$$

excluding biogenic activity in the ocean and oceanic chemistry. Eq. (2.7) has two free parameters, $1/\tau$ and b that can be evaluated by nonlinear optimization. We note that the input of Eq. (2.7) is a) the anthropogenic CO₂ emissions $\bar{n}_{tot}(t)$, b) the sea surface temperature term $\bar{S}_a(t)$, and c) the constant \bar{N}_0 .

The differential equation Eq. (2.7) can be solved numerically yielding $N_a(t)$. $\bar{N}_a(t_0)$ is used as initial condition for $t = t_0$. By the sum rule Eq. (2.3) the fluxes $n_s(t)$ and $n_b(t)$ are determined from $n_a(t) = dN_a(t)/dt$. Finally, by numerical integration of $n_s(t)$, and $n_b(t)$ the quantities $N_s(t)$ and $N_b(t)$ are obtained. The model results $n_a(t)$, $n_s(t)$, $n_b(t)$, $N_a(t)$, $N_s(t)$, and $N_b(t)$ can be directly compared with the observations for the period 1959 - 2013 [4]. For the earlier period 1850 - 1959 only $\bar{n}_a(t)$ and $\bar{N}_a(t)$ exist for a comparison.

3 THE DATA

Data on the carbon cycle are subdivided here into the periods 1850 to 1959, 1959 to 2013, and 2013 to 2150. For the period 1850 to 1959 only the estimations for anthropogenic CO₂ emissions and proxy data on the CO₂ content of the atmosphere are available [4],[18],[19],[1].

The year 1959 can be considered as the start of more systematic data acquisition on the carbon cycle. From this year on measurements, observations and estimations of carbon fluxes are reported [4],[2] as follows: Rate growth of atmospheric CO₂ concentration $\bar{n}_a(t)$, fossil fuel burning and cement production $\bar{n}_{fuel}(t)$, land use change such as deforestation $\bar{n}_{landuse}(t)$, ocean sink $\bar{n}_s(t)$, and transforming organic materials in the biosphere $\bar{n}_b(t)$. $\bar{n}_a(t)$ are obtained by direct measurements, $\bar{n}_{fuel}(t)$ from energy statistics, $\bar{n}_{landuse}(t)$ from data of land cover change, deforestation and from models. $\bar{n}_s(t)$ are based on the mean ocean sink obtained by observations in the 1990 decade and ocean models. $\bar{n}_b(t)$ is the residual of the other budget terms such as $\bar{n}_b = (\bar{n}_{fuel} + \bar{n}_{landuse}) - \bar{n}_a - \bar{n}_s$. The total anthropogenic emissions are $\bar{n}_{tot}(t) = \bar{n}_{fuel}(t) + \bar{n}_{landuse}(t) = \bar{n}_a(t) + \bar{n}_b(t) + \bar{n}_s(t)$. In the upper panel of Fig. 1 the graphs of $\bar{n}_{tot}(t)$ and the atmospheric CO₂ concentration are shown. The graphs of $\bar{n}_a(t)$, $\bar{n}_b(t)$, and $\bar{n}_s(t)$ are given in the upper right and in both lower panels of Fig. 2.

For future (here from the year 2014 on) global anthropogenic emissions in GtC/yr are estimated as six different scenarios $\bar{n}_{tot}(t)$ which end in the year 2100 [20]. They are shown in the upper left panel of Fig. 3. Because these emission scenarios end in the year 2100, we extended them until the year 2150 by tentatively assuming a decrease of anthropogenic CO₂ emissions. Decreasing emissions after 2100 suggest itself because the global coal reserves are estimated as roughly 1100 GtC [21] and the extreme scenario with an integrated value of ~ 2000 GtC [20] exceeds this value nearly twofold. Our tentative assumption after 2100 is a simple linear decrease of $\bar{n}_{tot}(t)$ to half the pertinent values of the year 2100 and is shown in the upper panel of Fig. 3.

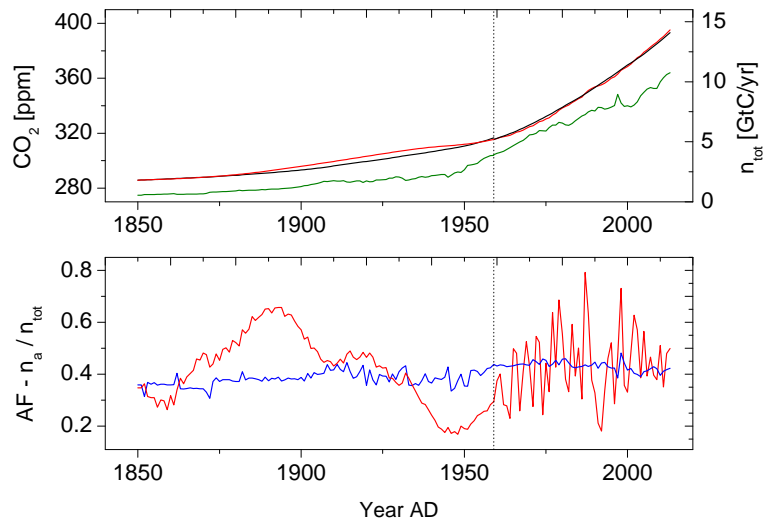


Fig. 1. (Color online) Atmospheric CO₂ (upper panel) and Airborne Fraction $AF = n_a(t) / \bar{n}_{tot}(t)$ (lower panel) versus the observations [4],[1] for the period 1850 - 2013. The vertical dotted line indicates the start of the full data set from AD 1959 on. Upper panel, left y-axis: Atmospheric CO₂ concentrations, model (black), observations (red), right y-axis: Anthropogenic emissions $\bar{n}_{tot}(t)$ (green). Lower panel: $AF = n_a(t) / \bar{n}_{tot}(t)$ (model blue, observations red)

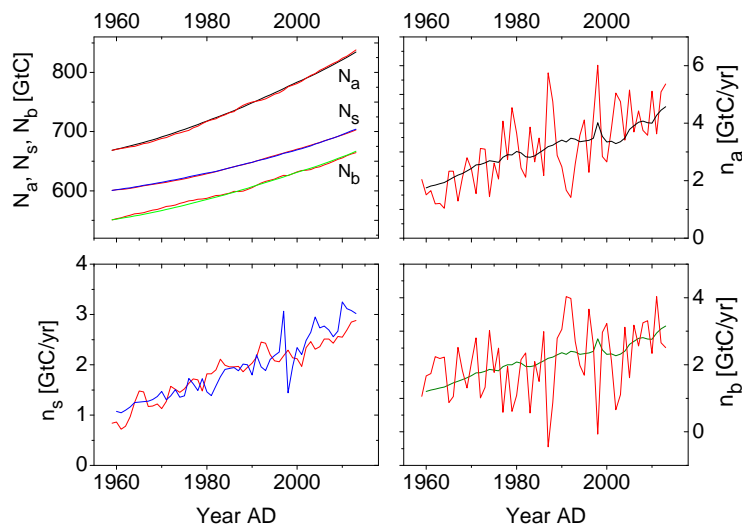


Fig. 2. (Color online) Time series of carbon fluxes $n(t)$ and their integrated values $N(t)$ versus the observations [4] for the period 1959 - 2013 (observations generally in red color). Left upper panel: Atmospheric carbon content $N_a(t)$ (black), oceanic carbon content $N_s(t)$ (blue), and biospheric carbon content $N_b(t)$ (green). $N_a(t)$ is identical with CO₂ (see Fig 1 upper panel) except for the factor 2.12. The time series $N_s(t)$ and $N_b(t)$ are shifted for clarity, their initial values of the year 1959 are not known. Right upper panel: fluxes into the atmosphere $n_a(t)$ (black). Left lower panel: fluxes into the ocean $n_s(t)$ (blue). Right lower panel: fluxes into the biosphere $n_b(t)$ (green)

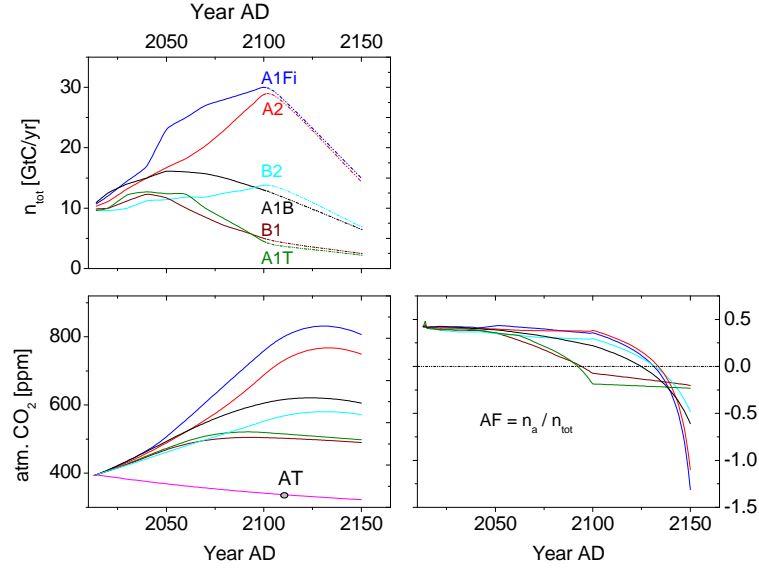


Fig. 3. (Color online) Experiments on atmospheric CO₂ in the future 2014 to 2150 AD by six carbon emission scenarios and, separately, a sudden stop of anthropogenic carbon emissions in 2013 AD. Upper panel: Anthropogenic carbon emission scenarios A1Fi, A2, B2, A1B, B1, and A1T given in [20] for 2013 to 2100 AD with the assumed continuations until 2150 AD (dotted lines). Left lower panel: Atmospheric CO₂ concentrations according to the six scenarios. The dashed curve is due to the separate experiment of a stop of anthropogenic carbon emissions in 2013 AD, the small grey square indicating the year of the Adjustment Time AT (half-life). Right lower panel: Airborne Fraction $AF = n_a(t) / \bar{n}_{tot}(t)$ of the six emission scenarios. The horizontal dotted line indicates $AF = 0$ where the sum of the CO₂ net-fluxes from atmosphere into ocean and biosphere equals the anthropogenic CO₂ emissions into the atmosphere

4 METHODS

The integration of a first order differential equation $dy/dt = f(t, y(t))$ such as Eq. (2.7) can be simply carried out by the explicit EULER technique [22].

$$y(t + \Delta t) = y(t) + \Delta t \cdot f(t, y(t)) \quad (4.1)$$

The use of more elaborate numerical methods, such as RUNGE-KUTTA, yields no substantially different results for Eq. (2.7). Using the time step of $\Delta t = 1$ year and the initial value $\bar{N}_a(t_0)$, Eq. (2.7) and Eq. (4.1) lead with $A_{i-1} = 1/(1 + b)[\bar{n}_{tot}(t_{i-1}) - 1/\tau(N_a(t_{i-1}) + \bar{S}_a(t_{i-1}) - \bar{N}_0)]$ to the following iteration ($i = 1, 2, \dots$):

$$n_a(t_i) = A_{i-1} \quad (4.2)$$

$$N_a(t_i) = N_a(t_{i-1}) + n_a(t_i) \cdot \Delta t \quad (4.3)$$

$$n_s(t_i) = \bar{n}_{tot}(t_i) - (1 + b) \cdot n_a(t_i) \quad (4.4)$$

$$n_b(t_i) = \bar{n}_{tot}(t_i) - n_a(t_i) - n_s(t_i) \quad (4.5)$$

$$N_s(t_i) = N_s(t_{i-1}) + n_s(t_i) \cdot \Delta t \quad (4.6)$$

$$N_b(t_i) = N_b(t_{i-1}) + n_b(t_i) \cdot \Delta t \quad (4.7)$$

The Airborne Fraction AF , defined as $AF = n_a(t) / \bar{n}_{tot}(t)$, is obtained from the result $n_a(t)$ of Eq. (4.2). Initial values of $\bar{N}_s(t_0)$ and $\bar{N}_b(t_0)$ for the iteration of Eqs. (4.6, 4.7) are not known. Thus these values are uncertain by an additive constant and the graphs $\bar{N}_s(t)$, $\bar{N}_b(t)$ have been arbitrarily shifted for clarity. As already mentioned, for the early period 1850-1959 only the observations $\bar{n}_{tot}(t)$, $\bar{n}_a(t)$ and $\bar{N}_a(t)$ are available. For the period 1959 - 2013 additionally $\bar{n}_s(t)$ and $\bar{n}_b(t)$ are provided. For the future 2013 - 2150 only scenarios of $\bar{n}_{tot}(t)$ are at hand.

The root of the added squared differences $d_a(t) = (\bar{N}_a(t) - N_a(t))^2$, $d_s(t) = (\bar{N}_s(t) -$

$N_s(t))^2$, and $d_b(t) = (\bar{N}_b(t) - N_b(t))^2$ yields a measure of the model accordance with the observations, for the period 1850 - 1959 as

$$G(\tau, b) = \sqrt{\sum_{t=1851}^{1959} d_a(t)} \quad (4.8)$$

and for the period 1959 - 2013 as

$$F(\tau, b) = \sqrt{\sum_{t=1960}^{2013} [d_a(t) + d_s(t) + d_b(t)]} \quad (4.9)$$

Conditional on the different data basis of the periods 1850 - 1959, 1959 - 2013, and 2013 - 2150 the iteration equations Eqs. (4.2) - (4.7) are solved separately as follows: Eqs. (4.2, 4.3) and the initial value $\bar{N}_a(1850) = 605.8$ GtC are used for the period 1851 - 1959 yielding G of Eq. (4.8). Eqs. (4.2 - 4.7) and the initial value $\bar{N}_a(1959) = 668.6$ GtC are used for the period 1960 - 2013 yielding F of Eq.(4.9). Finally, Eqs. (4.2, 4.3) and the initial value $\bar{N}_a(2014) = 842$ GtC are applied for the future period 2014 - 2150.

The model allows to evaluate the AT for the restoration of a stationary state after a CO₂ perturbation. We followed [7] by applying a CO₂ impulse of 100 GtC in the year 2010 and evaluated the model response as the time dependent CO₂ amount remaining from the impulse. The results of [7] and of our model are compared in Fig. 4. As an alternative evaluation of the AT, we assumed the anthropogenic CO₂ emission from 2013 on completely stopped and analyzed the decreasing atmospheric CO₂ from this time on. The result of this numerical experiment is shown in the left lower panel of Fig. 3.

5 PARAMETER ESTIMATION

The procedure of nonlinear optimization for estimation of the parameters τ and b in Eqs. (2.1, 2.2) minimizes the objective function $F(\tau, b)$ given by Eq. (4.9). This means that we restricted the nonlinear optimization to the period 1959 - 2013 of the more reliable measurements. We note that we could also have used a related objective function of the fluxes, instead of the integrated fluxes, as direct data. However, the large scatter of the fluxes leads the minimization very often to local minima. In contrast, minimizing the objective function for the integrated fluxes always yields unique global minima. For the minimizing procedure we applied the nonlinear SIMPLEX method [23].

The results of the optimization are depicted in Table 1 and show no essential differences by including or excluding the temperature term of Eq. (2.6). In the following we therefore refer generally to the parameter values of the optimization that exclude the temperature term.

The time series of the model together with the pertinent CDIAC observations are shown in Fig. 1 for the years 1850 to 1959 and in Fig. 2 for the years 1959 to 2013. We mention that the values of the parameter τ in Table 1 agree well with an estimation of 81.4 years given in [13] for the oceanic carbon uptake. The value of parameter b of Eq. (2.2) goes well together with measurements of increases in yield for food plants under rising atmospheric CO₂ concentrations [16] (see also 6. Results).

Table 1. Model parameters τ, b of Eqs. (2.1, 2.2) from nonlinear minimization. $F(\tau, b)$ of Eq. (4.9) is the minimized objective function that gives the difference between model and CDIAC observations in the period 1959 to 2013. $G(\tau, b)$ of Eq. (4.8) gives the difference of model and observations in the period 1850 to 1959. Row 1 includes the temperature term $\bar{S}_a(t)$ of Eq. (2.6), row 2 excludes it from the optimization

τ [yr]	b	$F(\tau, b)$	$G(\tau, b)$	$\bar{S}_a(t)$
80.3	0.697	27.7	58.6	$15.9 \cdot T(t)$
84.0	0.697	28.1	53.3	0

6 RESULTS

The agreement of the integrated quantities $N_a(t)$, $N_s(t)$, and $N_b(t)$ with the observations is nearly perfect, differences are hardly detectable by eye (see Fig. 1 and Fig. 2). The yearly fluctuations of AF (see lower panel of Fig. 1) and the fluxes $n(t)$ (see Fig. 2) are not simulated as perfectly as the integrated quantities, only the average courses over several years show good agreement. These fluctuations are probably caused by natural variations, as for instance varying intensity of the photosynthesis in the seasons or El Niño, which can not be considered in our simple model. Noise in the observations data could be an additional cause. As a main conclusion the overall perfect agreement of the model with the observational data indicates that model linearity is a valid approximation at least for the long period of the last 163 years of observations. The following details confirm this:

- (a) The two parameters of our model are evaluated by minimizing the objective function $F(\tau, g)$ of Eq. (4.9) for the period 1959 - 2013. Hence, one could expect that these parameters would not agree as well with the observations of the longer early period 1850 - 1959. However, this is not the case (see Fig. 1), which indicates the robustness of our linear model over periods not considered by the parameter estimation.
- (b) Gloor et al. [13] found a similar value of 81.4 years for the time constant in Eq. (2.1) for the net-flux of atmospheric CO₂ into the ocean.
- (c) Parry et al. [16] report that a wide variety of food plants show nearly linear increases in yield from 0.02% to 0.06% per 1 ppm rising atmospheric CO₂ until an atmospheric CO₂ concentration of 750 ppm. This fits well with our model: From the total anthropogenic CO₂ emissions of ~ 10.5 GtC/yr in the year 2013 our model yields ~ 4.5 GtC/yr or $4.5/2.12 = 2.2$ ppm/yr atmospheric CO₂ increase and ~ 3 GtC/yr absorbed in roughly equal amounts by ocean and biosphere (see Fig. 2). The total carbon content of the biosphere is roughly 4000 GtC [24]. If one assumes roughly equal increase for the total biosphere as for food plants by rising atmospheric CO₂, as reported in [16], 2.2 ppm more CO₂ in the year 2013 cause a carbon increase of the biosphere between $2.2 \cdot 0.0002 \cdot 4000 = 1.8$ GtC

and $2.2 \cdot 0.0006 \cdot 4000 = 5.3$ GtC. The above mentioned ~ 3 GtC of our model fit with these values.

The discussion about man-made global warming in mind we think that an estimation of the future increase of atmospheric CO₂ by our linear model could be useful and valid. For this purpose we postulate that the linearity of our model remains at least for the next 100 years. This seems reasonable because in spite of an atmospheric CO₂ increase of about 40% from the year 1850 until present an increase of the AF cannot be seen in the CDIAC data (see lower panel of Fig. 1, red curve and the abstract of [13]).

Under the assumption of future model validity and depending on the emission scenarios given in [20] until 2100 AD as well as the assumed emission scenarios after 2100 AD until 2150 AD (see Fig. 3, upper panel), our model gives the results depicted in the lower panels of Fig. 3: The atmospheric CO₂ content will not exceed a maximum between ~ 500 to ~ 800 ppm (see left lower panel of Fig. 3). The AF remains constant until 2050 AD and does decrease later steadily for all scenarios. At maximum atmospheric CO₂ concentration the atmospheric net-flux n_a and with it the $AF = n_a(t) / \bar{n}_{tot}(t)$ change sign because the sum of the net-fluxes of atmospheric CO₂ into the ocean and the biosphere is smaller than the emitted anthropogenic CO₂ (see right lower panel of Fig. 3).

The AT of the atmospheric CO₂ has been calculated with our model based on two different numerical experiments. In the first experiment we stopped the anthropogenic CO₂ emissions in the year 2013. As a consequence, the atmospheric CO₂ concentration begins to decrease until the natural equilibration will be accomplished. The half-life $t_{1/2}$ of this process $N_a(t_{1/2}) = [N_a(2013) + N_a(\infty)]/2$, as defined for the AT, is reached in about 100 years. In the left lower panel of Fig. 3 the CO₂ curve of this experiment is shown and the year of the AT indicated by a grey disk.

A different experiment was discussed in [7] comparing the response of 15 known comprehensive carbon cycle models on a 100 GtC impulse charged in the year 2010 and after

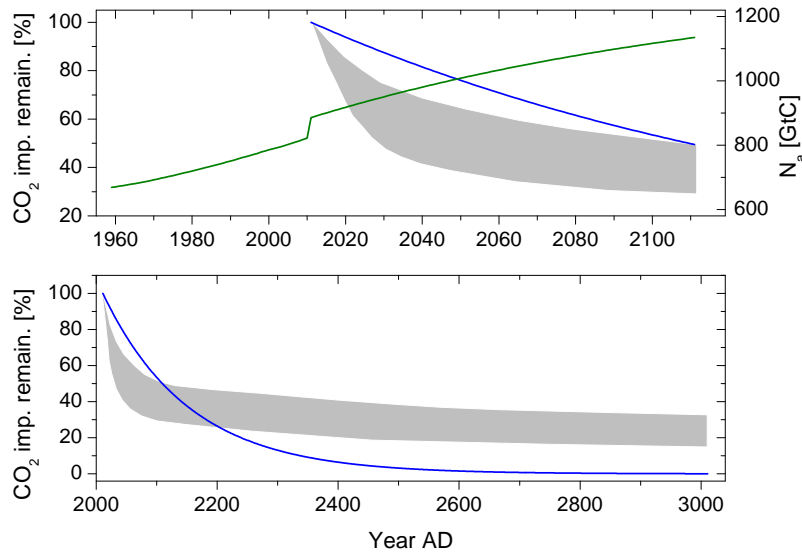


Fig. 4. (Color online) Remaining atmospheric CO₂ as implication of a numerical experiment discussed in [7] which applies a 100 GtC impulse in the year 2010 and keeps from then on the anthropogenic carbon emissions constant at 10 GtC/yr. Upper panel covering the period 1960 to 2150, left y - axis: CO₂ remaining by our model (blue) and from 15 known comprehensive models discussed in [7] (grey shaded area). Right y - axis: Atmospheric carbon content $N_a(t)$ calculated with our model which shows the impact of the 100 GtC impulse (green). Lower panel: remaining CO₂ over the longer period until 3000 AD, our model (blue), 15 comprehensive models (grey shaded area)

the year 2010 constant anthropogenic carbon emissions on the 2010-year-level of 10 Gt/yr. The response here is the percentage of carbon remaining from the impulse, obtained from the difference of the atmospheric carbon including and excluding the impulse. The AT of this experiment with our model results in 100 years, equal to the result of the above mentioned first experiment.

Fig. 4 shows the comparison of our linear model with the results published in [7]. During the first 100 years after the impulse the decrease of the response of our model lies above the upper limit of the 15 comprehensive models but after 2200 years distinctly below their lower limits. In a far distant future the impulse-response of our model approaches zero whereas all responses of the comprehensive models do not fall below ~20%. The difference may stem from the Revelle effect, included in the elaborate models, a resistance to absorbing atmospheric CO₂ by the ocean due to bicarbonate chemistry.

However, there exists so far no evidence for the Revelle effect in the CDIAC data. Gloor et al. [13] emphasize similarly in the abstract as *"claims for a decreasing long-term trend in the carbon sink efficiency over the last few decades are currently not supported by atmospheric CO₂ data and anthropogenic emissions estimates"*. Thus, such effects are at present hypothetical. This could be related to (1) uncertainties in current estimates of emissions, (2) noise in the AF due to natural climate variability, or (3) still unknown biogenic effects, for instance a higher downward flow of organic carbon in the deeper ocean over many decades or centuries [25].

7 CONCLUSIONS

Our linear model with only two parameters gives perfect agreement with the major observation components of the global carbon cycle provided by [4] as are the integrated carbon rates of the atmosphere $\bar{N}_a(t)$, the ocean $\bar{N}_s(t)$, and the biosphere $\bar{N}_b(t)$. Also the agreement with

the observed yearly carbon net-fluxes $\bar{n}_a(t)$, $\bar{n}_s(t)$, and $\bar{n}_b(t)$ is on average perfect. That the fluctuations of these fluxes are larger than those of the model is apparently due to the non-inclusion of natural variations (such as different strength of the photosynthesis in the seasons and El Niño etc.). Our model gives no indication that the change of sea surface temperatures since the beginning of industrialization has an appreciable influence on the anthropogenically forced carbon cycle.

The linearity of our model and increases in yields of food plants due to rising atmospheric CO₂ concentrations are confirmed by further authors [13],[16],[11],[12]. The agreement of our simple model with the observations [9],[26] indicates that we are far from the influences of nonlinearities or the Revelle effect. As a consequence, we expect no substantial deviations from the linear model results at least for the next 100 years.

With regard to the actual discussion about an assumed dangerous climate change by anthropo-genic CO₂ emissions in future, our most important model result yields the impact of anthropogenic CO₂ emission scenarios given in [20] until the year 2100 AD (completed by own tentative continuations from 2100 to 2150 AD). For all scenarios, between 2080 to 2140 AD atmospheric CO₂ concentrations up to at most ~500 until ~800 ppm occur. These future turning points of the atmospheric CO₂ content take place 30 years after the pertinent emissions maxima. The maximum of 800 ppm atmospheric CO₂ concentration seems to be safe because the pertinent emission scenario assumes burning a twofold of the actual known coal reserves worldwide. After the apogee of atmospheric CO₂ concentration predominantly the ocean and the biosphere will be the sinks of future CO₂ emissions of mankind.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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